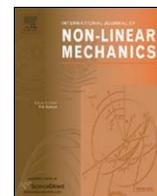




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# International Journal of Non-Linear Mechanics

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## Editorial

### Introduction to the special issue on waves in non-linear solid mechanics

The propagation of large amplitude waves has fascinated scientists for hundreds of years. The mathematical theory of non-linear waves is rooted in attempts to solve specific, concrete problems, many of them related to the propagation of water waves. The most significant research activity on this problem can be traced back to the 19th century, including the classic works of Stokes, Lord Rayleigh, Korteweg and de Vries, Boussinesq, and many others, all the way to the numerical discovery of the soliton in the early 1960s by Zabusky and Kruskal in their study of collisionless plasma. The subject of non-linear waves is better established and has been studied in greater depth in fluids than in solids, probably because they are much more difficult to observe in the latter than in the former. John Scott Russell was able to jump on a horse to follow and even overtake the solitary water wave traveling on the Union Canal in Edinburgh, Scotland; in contrast, seismic waves travel typically at several kilometers per second! Moreover, the amplitude of a motion in a solid is bounded by the elastic limit. In effect, the vast majority of scientific and technological applications in solid acoustics takes place in the context of linear wave propagation.

However, non-linearity can be introduced in several ways. For example, it can be assumed that an infinitesimal (linearized) wave propagates in a finitely (non-linearly) deformed solid; the resulting “incremental” equations of motion provide a framework for the non-destructive evaluation and characterization of pre-strained solids. Or, it can be assumed that the solid is sufficiently “soft” that it is able to support second-order wave effects without plastic yield.

In fact, wave theories in non-linearly elastic solids are beginning to score important successes in many fields of science and engineering. For example, ultrafast scanners provide a powerful tool for detecting shear wave propagation in biological soft tissues and in phantom gels in transient elastography experiments, such as those conducted at the *Laboratoire Ondes et Acoustique* in Paris; this opportunity opens up new horizons for the non-invasive biomedical imaging of tumoral masses. The Geophysics Group at the *Los Alamos National Laboratory* has observed what they called “dynamic non-linear elastic behavior” in earth materials, probably due to the presence of “soft regions”; this observation has opened up new perspectives in experimental, numerical, and theoretical geophysics and in the non-linear spectroscopy of materials.

These two examples demonstrate that new skills and new methods are required in the modeling of waves in non-linear solids, if only to accompany the development of related technologies. In this volume we present some of these theoretical advances. We have collected various papers from some leading experts, hoping to get a (partial) snapshot of the current activity in the field of waves in non-linear media.

The works collected here consider non-linear effects from different points of view. Some of them are concerned with the strong, but fruitful approximation of small-amplitude motions superimposed on large strains. In another class of approximation, some authors consider the theory of weakly non-linear waves in elastic solids. Others tackle the fully non-linear equations of motion. In such a way it was possible to collect a wide range of mathematical methods and modeling approaches, giving a broad overview of the topic of waves in non-linear solids.

With respect to the incremental approach, the articles by Murphy and Destrade, Edmondson and Fu, and Deschamps and Huet are most innovative. Murphy and Destrade's investigation of the influence of compressibility on incremental surface waves in pre-stressed solids generalizes some results obtained recently in the special case of incompressible materials. On the other hand, Edmondson and Fu succeed in the difficult task of incorporating internal constraints in the Stroh formulation of waves in pre-stressed materials: here the constraint may be isotropic (such as incompressibility) or even anisotropic (such as inextensibility). The Deschamps and Huet in-depth study of inhomogeneous Rayleigh waves and associated wavefronts is remarkable because it leads to a wonderful coincidence between theory and experimental results. This paper is not directly placed within the context of incremental waves *per se*, but we included it because there is a well-established correspondence between results derived for incremental waves in strain-induced anisotropy and results derived for linear waves in crystal anisotropy; that correspondence deserves to be investigated further in order to extend those beautiful results.

Turning now to other mathematical methods, the article by Domanski and Norris shows how to derive the evolution equations at quadratically non-linear level for the amplitudes of quasi-longitudinal and quasi-transverse waves propagating in arbitrary anisotropic media. At the same (quadratic) level of approximation for weakly non-linear waves, Domanski investigates the influence of incompressibility: his study follows from the experimental observations that soft gels and biological soft tissues are essentially incompressible, and that the equations governing the propagation of weakly non-linear waves in those solids must be modified accordingly. Dai and Li develop a method based on dynamical systems theory to study traveling waves in hyperelastic rods, with an application to Mooney–Rivlin materials that may be generalized to more complex constitutive equations. Similarly, Christov and Jordan examine shocks and traveling waves in a taut string, using a powerful mix of qualitative and numerical methods. Rodrigues Ferriera studies the superposition of finite-amplitude shear waves propagating in arbitrary directions in some non-linear elastic materials subject

to a large homogeneous pre-strain; she shows that for appropriate choices of the polarization and the propagation directions, these waves are exact solutions for the class of Mooney–Rivlin materials.

The remaining papers are more related to modeling issues, and they demonstrate the importance and actuality of wave problems in the framework of non-linear solids. Ogden presents a general theory for the derivation of the incremental equations of motion for a deformed, electromagnetic-sensitive elastic solid; here there are two combined types of anisotropy due to the multi-physical coupling: an anisotropy induced by the large pre-deformation and an anisotropy due to the presence of a time-independent electromagnetic field. Rogerson and Prikazchikov use long-wave expansions to study generalizations of bending and extension for waves in prestressed plates. Porubov and Maugin go up to cubic non-linearities in order to describe the propagation of longitudinal waves in some seismic media. Pucci and Saccomandi show how to model the classic Melde experiment in the framework of non-linear elastodynamics; they use the full non-linear equations to reveal some parametric resonance effects. Durickovic, Goriely and Saccomandi show how to model a non-linear elastic rod in order to display the propagation of the remarkable class of solitary compact waves. Darbha and Rajagopal consider the modeling of unsteady motions in a material that *degrades*, for example, because of environmental oxidation; their paper provides a concrete example where chemistry meets mechanics. Kiernan, Cui and Gilchrist devote their paper to stress waves in functionally graded foams: here, a finite element model of wave propagation phenomena is used to understand the potential of such materials for cushioning structures.

We are delighted to have been able to put together such a collection of high quality articles on waves in non-linear solid mechanics. We are also happy to dedicate this volume to our mutual friend, Professor Philippe Boulanger, on the occasion of his retirement from the Département de Mathématiques at the Université Libre de Bruxelles. Philippe's work on finite-amplitude waves in deformed elastic solids has been appreciated worldwide for its elegance and also because it is the natural continuation of the classic works of Fritz John, Ray Ogden, Mike Hayes, Peter Currie, the Italian school of Signorini on Hadamard materials, and others.

We hope to have served all the solid mechanics community in editing this volume. We thank the editorial board of the International Journal of Non-Linear Mechanics to have granted us this opportunity, with special thanks to Ray Ogden for his superb editorial support.

#### **Bibliographical note on Philippe Boulanger on the occasion of his retirement, by M.A. Hayes**

As Philippe Boulanger retires from his position as a Professeur de Mécanique at the Département de Mathématique, Université Libre de Bruxelles, it seems fitting to dedicate this special issue to him, because so many of his contributions have touched and influenced research in the field of wave propagation in solids.

Philippe Boulanger has actively participated in the development of mechanics both nationally and internationally. More than 30 years ago he was a founder member of the International Society for the Interaction of Mathematics and Mechanics (and he is currently an elected member of its Executive Committee). He has been a member of the National Belgian Committee for Theoretical and Applied Mechanics since 2001. He was elected President in 2004 and has been active in the organization of its congresses and its Graduate School in Mechanics. From 2004 to 2008 he represented Belgium in the General Assembly of the International Union of Theoretical and Applied Mechanics. He was pleased with the award of the IUTAM Symposium on "Analysis and simulation of human movement" to be

held in Leuven in 2010, the first such IUTAM symposium in Belgium since 1973.

Following a brilliant school career, he entered ULB in 1966 to study Physical Science and passed all his university examinations at the highest level. He was awarded the Degree of Doctorat en Sciences with the grade: "La plus grande distinction, avec les félicitations du jury". He then moved swiftly up the academic ladder from the time he joined the staff as assistant in 1971 until he was appointed full professor in 1993. The bulk of his teaching is on analytical mechanics, continuum mechanics, linear and non-linear wave propagation. He has a well-deserved reputation as a clear lecturer who takes great pains with his presentation and content, and he is much appreciated by his students and his audience at national and international conferences.

Philippe Boulanger has been very active in research since he started in 1970 when, at the age of 22, he co-authored his first paper. His list of publications, which follows, shows the breadth and scope of his studies. His early work was in photoelasticity, in which he has retained an interest to the present time. He has made valuable insights into magneto-optical, electro-optical and photoelastic effects in a variety of media, with a particular interest in the Faraday and Hall effects. Later, these earlier studies in electromagnetic effects informed his researches in wave propagation in elastic and viscoelastic solids and viscous fluids. He is equally well at home dealing with anisotropic and isotropic media. He is expert in non-linear waves and is always alert to the possibility of finite amplitude waves in different systems. Wave propagation—both finite amplitude and infinitesimal amplitude—in finitely deformed elastic media was the subject of many studies. He is adept in using bivectors in the description and study of inhomogeneous plane waves and he exposed these connections in the 1993 monograph *Bivectors and Waves in Mechanics and Optics*, co-written with Michael Hayes. In particular, he studied infinitesimal amplitude waves in elastic crystals and gave a very simple way of determining bounds on wave speeds in cubic, orthorhombic, tetragonal and hexagonal crystals and, in a companion study, of determining the acoustic axes in these crystals. Kinematics of deformation is another area where he made very valuable contributions, with the study of unsheared pairs of infinitesimal material line elements, the determination of the pair of elements suffering the maximum shear in a deformation, the consideration of unsheared triads of infinitesimal material line elements, and the extended polar decomposition of the deformation gradient related to unsheared triads. He is currently completing a major study on finite amplitude inhomogeneous waves in classes of finitely deformed isotropic elastic materials, together with Elizabeth Rodrigues Ferreira, who has just finished her PhD under his direction.

The Belgian Academy of Sciences awarded Professor Boulanger the Agathon de Potter Prize for original research in Mathematics during the 3-year period 1976–1978, and the Georges Van Der Linden Prize for original research in Physics, in particular electromagnetic wave propagation, during the period 1989–1992. In his youth Philippe Boulanger excelled in Mathematics and Music. He decided on a career in Science in place of a career as a concert pianist but continues to play the piano and has often entertained fellow attendees at conferences with his exquisite playing. There is another side to his playing—he has been known to accompany young musicians in Dublin playing for their junior music examinations. Because he is very fond of animals, particularly cats, an animal support group calls on him to accompany their performers in their annual fund raising concert. His wife Violeta has been wonderfully supportive of Philippe and his work. My wish for them is that they have many more happy years ahead of them.

Michael Hayes,  
University College Dublin.

PS. A word about the Boulanger–Hayes collaboration. It has been a

real pleasure for me to work with Philippe. We met in 1987 following a letter from Robin Knops. To date we have produced 55 joint articles, a book, and some book chapters. From the beginning we adopted the three rules which Hardy and Littlewood drew up for their own very successful collaboration: (i) authors' names are always in the same order; (ii) neither author is expected to read within a specified time what the other sends; (iii) what one author sends to the other need not be correct.

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M. Destrade\*

*School of Electrical, Electronic, and Mechanical Engineering,  
University College Dublin, Belfield,*

*Dublin 4, Ireland*

*E-mail address: [destrade@imm.jussieu.fr](mailto:destrade@imm.jussieu.fr)*

G. Saccomandi

*Dipartimento di Ingegneria Industriale,*

*Università degli Studi di Perugia,*

*06125 Perugia, Italy*

13 February 2009

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\*Corresponding author