Chapter 1

First Year

1.1 MA133-1 Algebra, Sem I

Textbook: *Algebra & Geometry: An introduction to University Mathematics* by Mark V. Lawson.

Continuous Assessment: Six online homeworks which count for 25% of the module MA131.

1.1.1 Matrix Algebra

1. [Linear transformations of the plane]
   Decide which of the following functions $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ are linear and, for those that are not, give an example to demonstrate non-linearity.

   (a) $f(x, y) = (x^2, y^2)$.

   (b) $f(x, y) = (2x + 3y, 4x - y)$.

   (c) $f(x, y) = (2x + 3y + 1, 4x - y)$.

   (d) $f(x, y) = (3xy, x - y)$.

   (e) $f(x, y) = (0, 0)$.

   (f) $f(x, y) = (y, x)$.

2. [Linear transformations of the plane]

(a) Let $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that sends $(1,0) \mapsto (2,3)$ and $(0,1) \mapsto (3,-1)$. Evaluate $f(-1,4)$ and then find a general formula for $f(x,y)$.

(b) Let $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that rotates the plane about the origin through a clockwise turn of 90°. Evaluate $f(-1,4)$ and then find a general formula for $f(x,y)$.

(c) Let $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that reflects in the line $y = x$. Evaluate $f(-1,4)$ and then find a general formula for $f(x,y)$.

(d) Determine matrices that represent each of the linear functions in (a), (b) and (c).

3. [Matrix addition]
   Evaluate

(a) $\begin{pmatrix} 1 & 2 \\ -5 & 3 \end{pmatrix} + \begin{pmatrix} -2 & 0 \\ 4 & 1 \end{pmatrix}$,

(b) $\begin{pmatrix} 1 & 2 & 2 \\ -5 & 3 & 0 \\ 1 & 0 & -1 \end{pmatrix} + \begin{pmatrix} -2 & 0 & 7 \\ 4 & 1 & -6 \\ 0 & 0 & 5 \end{pmatrix}$,

(c) $2\begin{pmatrix} 3 & 2 & 1 \\ -2 & 2 & 1 \end{pmatrix} - \begin{pmatrix} 2 & 1 & 3 \\ 5 & 2 & 6 \end{pmatrix}$.

(d) Determine the matrix representing the linear function $f(x,y)+g(x,y)$ for $f(x,y) = (x + 2y, -5x + 3y)$ and $g(x,y) = (-2x, 4x + y)$.

(*cf.* [Lawson], Sec. 8.1)

4. [Matrix multiplication]

(a) Let $f(x,y) = (x+2y, 3y-x)$ and $g(x,y) = (y-2x, x+y)$. Determine a formula for the composite transformation $g \circ f\colon \mathbb{R}^2 \to \mathbb{R}^2, (x,y) \mapsto g(f(x,y))$.

(b) Evaluate the matrix product BA where

$$B = \begin{pmatrix} -2 & 1 \\ 1 & 1 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}.$$

5. [Matrix multiplication]
Let

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}, \quad D = \begin{pmatrix} 4 & -1 & 2 \end{pmatrix}.$$

Decide which of the following arithmetic expressions can be evaluated and evaluate those that can be.

(i) DB, (ii) BD, (iii) AC, (iv) CA, (v) BD,
(vi) DB, (vii) A^2, (viii) C^2, (ix) CAB.

6. [Inverse matrix]
Let $A = \begin{pmatrix} 3 & 1 \\ 1 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 5 \\ 1 & 8 \end{pmatrix}$.

(a) Calculate A^{-1} and B^{-1}.

(b) Verify that $(AB)^{-1} = B^{-1}A^{-1}$.

(c) Use A^{-1} to solve the system of equations

$$\begin{aligned} 3x + y &= 13 \\ x - 2y &= 2 \, . \end{aligned}$$

(d) Find a 2×2 matrix X such that $AX = B$.

(*cf.* [Lawson] Example 5.5.7)

7. [Inverse matrix]
Determine a value for x for which the matrix $A = \begin{pmatrix} 5 & x \\ 2 & 4 \end{pmatrix}$ has no inverse.

8. [Inverse matrix]

(a) Consider the matrices

$$A = \begin{pmatrix} 3 & 2 & 6 \\ 4 & 3 & 4 \\ 4 & 2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & -6 & 10 \\ -4 & 15 & -12 \\ 4 & -2 & -1 \end{pmatrix}.$$

Calculate the product AB.

(b) A brewery produces Brown Ale, Dark Brown Ale and Porter. The following table summarizes the amount of malt, hops and yeast used to produce one keg of beer together with the total amount of these resources available per day.

Resource	Brown Ale	Dark Brown Ale	Porter	Daily available
Yeast	3 kg	2 kg	6 kg	250 kg
Hops	4 kg	3 kg	4 kg	220 kg
Malt	4 kg	2 kg	3 kg	170 kg

i. Let x, y, z be the number of kegs of Ale, Brown Ale and Porter produced daily. Write down a system of three linear equations which hold precisely when all three resources are fully used.

ii. Use (a) to find the values of x,y,z that ensure that the daily supply of hops, malt and yeast are fully used.

9. [Image of lines under linear transformations]
Let $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the mapping $f(x, y) = (x - 2y, 2x + y)$.

(a) Find the image of the line $2x + y = 4$ under f. *i.e.* Find the equation of the image.

(b) Find the pre-image of the line $2x + y = 4$ under f. *i.e.* Find the equation of the pre-image.

10. [Composite linear transformations]
Let $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ be reflection in the line $y = x$ and let $g\colon \mathbb{R}^2 \to \mathbb{R}^2$ be reflection in the line $y = 0$.

(a) Find the matrices of f and g.

(b) Find the matrix of the composite transformation $g \circ f\colon \mathbb{R}^2 \to \mathbb{R}^2$.

(c) Find the matrix of the composite transformation $f \circ g\colon \mathbb{R}^2 \to \mathbb{R}^2$.

1.1.2 Eigenvalues of 2×2 matrices

1. [Calculating MA133-1/determinants]
Calculate the determinants of the following matrices.

$$A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}, B = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}, C = \begin{pmatrix} 4 & 5 \\ 2 & 3 \end{pmatrix}, D = \begin{pmatrix} 6 & 8 \\ 4 & 5 \end{pmatrix}.$$

(*cf.* [Lawson], Exercise 8.4.1.)

2. [Determinants & area]
   Calculate the area of the parallelogram with vertices $U = (0,0), V = (2,3), W = (3,4), X = (5,7)$.

   (*cf.* [Lawson], Theorem 9.3.2)

3. [Determinants & invertibility]
   Find all values of x for which the matrix $A = \begin{pmatrix} 12 & x \\ x & 18 \end{pmatrix}$ is not invertible.

4. [Eigenvalues and eigenvectors]
   Find the eigenvalues and corresponding eigenvectors of the matrix $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$.

   (*cf.* [Lawson], Example 6.8.10)

5. [Eigenvalues and eigenvectors]
   Let A be the 2×2 matrix representing reflection in the line $y = -x$. Find all eigenvalues and corresponding eigenvectors of A.

6. [Eigenvalues and eigenvectors]
   Determine a value of x for which the matrix $A = \begin{pmatrix} 0 & 1 \\ x & 0 \end{pmatrix}$ has no eigenvvectors.

7. [Matrix diagonalization]
   Let $A = \begin{pmatrix} -3 & 5 \\ -2 & 4 \end{pmatrix}$.

   (a) Find the eigenvalues and corresponding eignevectors of A.
   (b) Find an invertible matrix E such that $E^{-1}AE$ is diagonal.

   (*cf.* [Lawson], Example 8.6.10)

8. [Matrix diagonalization]
   Let $A = \begin{pmatrix} 7 & -12 \\ 2 & -3 \end{pmatrix}$.

   (a) Find the eigenvalues and corresponding eignevectors of A.
   (b) Find an invertible matrix E such that $E^{-1}AE$ is diagonal.

(c) Hence, or otherwise, calculate A^9.

(*cf.* [Lawson], Example 8.6.10)

9. [Markov processes]
The Jalopy Car Rental Company has offices in Galway and Cork. Each week 90% of the cars hired out in Galway are returned to Galway and the other 10% are returned to Cork. Of the cars hired out in Cork 95% remain in Cork and 5% are returned to Galway. The company initially has 80 cars in Galway and 60 cars in Cork. How many cars will there be in each place

   (a) in one week?
   (b) intwo weeks?
   (c) in the long term?

10. [Markov processes]
A school of 1000 students is quarantined due to the presence of a contagious disease. Each day 20% of those that are ill become well, and 30% of those that are ill become well. Initially nobody is ill. How many students are ill after

   (a) after 1 day?
   (b) after 2 days?
   (c) in the long run?

1.1.3 Number Theory

1. [Clock arithmetic]
Calculate the following.

   (a) $6 + 9\ mod\ 12$
   (b) $6 - 9\ mod\ 12$
   (c) $6 \times 9\ mod\ 12$

   (*cf.* [Lawson], Sec. 5.4)

2. [Clock arithmetic]
Decide which of the following inverses exist, and calculate those that do.

(a) 5^{-1} mod 9

(b) 5^{-1} mod 10

(c) 5^{-1} mod 11

(d) 5^{-1} mod 12

(e) 5^{-1} mod 15

(f) 5^{-1} mod 16

(*cf.* [Lawson], Sec. 5.4)

3. [ISBN]
One of the following numbers is the ISBN for *La drôle d'histoire du Finistère*. The other contains an error. Which is which?

   (a) $2 - 9510 - 5011 - 2$

   (b) $2 - 9150 - 5011 - 2$

4. [ISBN]
Determine the third digit of the ISBN number 3-5?0-90336-4.

5. [Euclidean algorithm]
Use the Euclidean algorithm to find integers x and y such that $gcd(a, b) = ax + by$ for each of the following pairs of numbers.

   (a) 112, 267.

   (b) 242, 1870.

   (*cf.* [Lawson], Sec. 5.2)

6. [Euclidean algorithm]
You have an unlimited supply of 3-cent stamps and an unlimited supply of 5-cent stamps. By combining stamps of different values you can make up other values: for example, three 3-cent stamps and two 5-cent stamps make the value 19 cents. What is the largest value you cannot make? Hint. You need to show that the question makes sense.

   (*cf.* [Lawson], Sec. 5.2)

7. [Euclidean algorithm & invertible numbers]

(a) Use the Euclidean algorithm to find the inverse of 14 modulo 37.

(b) The enciphered message

$$HVVH$$

was produced by applying the enciphering function

$$f_E \colon \mathbb{Z}_{37} \to \mathbb{Z}_{37}, x \mapsto 14x + 20$$

to single letter message units over the 37-letter alphabet

$$0, ..., 9, A = 10, B = 11, \dots, Z = 35, \_ = 36 \ .$$

i. Determine the corresponding deciphering function.

ii. Decipher the message.

8. [Euclidean algorithm & invertible numbers]
The enciphered message

$$AEF$$

was produced by applying the enciphering function

$$f_E \colon \mathbb{Z}_{26} \to Z_{26}, x \mapsto 11x + 4$$

to single letter message units over the alphabet $A = 0, B = 1, \dots, Z = 25$.

(i) Use the Euclidean algorithm to find the inverse of 11 modulo 26.

(ii) Determine the corresponding deciphering function.

(iii) Decipher the message.

9. [Euler Phi function]

(a) Factorise 270 as a product of primes.

(b) Calculate $\phi(270)$.

(c) Determine the number of integers from 1 to 270 that are coprime to 270.

(d) How many invertible elements are there in $\mathbb{Z}_{270}$?

10. [Euler Phi function]

(a) Factorise 1800 as a product of primes.

(b) Calculate $\phi(1800)$.

(c) Determine the number of integers from 1 to 1800 that are coprime to 1800.

(d) How many invertible elements are there in $\mathbb{Z}_{1800}$?

1.2 MA133-2 Calculus, Sem I

Textbook: *Calculus. Early Transcendentals* by James Stewart.

Continuous Assessment: Six online homeworks which count for 25% of the module MA131.

1.2.1 Functions & Graphs

1. [Linear functions]
   As dry air moves upwards it expands and cools. The temperature T of the air is a linear function of the height h. The ground temperature is $20°$C and the temperature at a height of 1km is $10°$C.

   (a) Express T (in °C) as a function of h (in km).

   (b) What is the temperature at a height of 2.5km?

   (c) At what height will the air temperature be $-15°$C?

   (d) Sketch the graph of the function $T(h)$. What does the slope of the graph represent?

   (e) The expression for T clearly depends on location and time since ground temperature varies with these two factors. Two Met Eireann weather balloons pass above Galway on a given dry day and measure the air temperature to be $-5°$C at a height of 3km and $10°$C at a height of 1.5km. What is the temperature in Galway on that day?

   (*cf.* [Stewart], Sec. 1.2, Example 1)

2. [Polynomial functions]
   A short-term economic model assumes that a country's GDP G (in €100

billion) at time t (in years) can be expressed as a quadratic polynomial $G = at^2 + bt + c$.

Initially G has a value of 20. At 6 months the value of G is 29 and at 1 year the value of G is 40.

(a) Determine the values of the constants a, b, c.

(b) Evaluate G for $t = 0, 1, 2, 3, 4, 5$ years.

(c) Sketch the graph of $G(t)$ over the range $0 \leq t \leq 5$.

(d) The *growth* over a time interval starting at time t_1 and ending at time t_2 is defined to be the number

$$\frac{G(t_2) - G(t_1)}{G(t_1)} .$$

Calculate the predicted growth for the interval from $t_1 = 1$ to $t_2 = 2$ years.

(e) The EU defines a country to be *in recession* at time t if its growth is negative for the interval from $t_1 = t - 0.25$ to $t_2 = t$ years and also for the interval from $t_1 = t - 0.5$ to $t_2 = t - 0.25$ years. What does the economic model predict about recession at time $t = 30$ months?

3. [Rational functions]
For each of the formulae

$$(i)\ f(x) = \frac{1}{1-x}\,, \quad (ii)\ f(x) = \frac{x}{1-x}\,, \quad (iii)\ f(x) = \frac{x-2}{1-x}$$

(a) determine all those real numbers x for which $f(x)$ is not defined.

(b) evaluate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

(c) sketch the graph of $f(x)$.

Then,

(d) for $x = 0.8$, evaluate the sum $x + x^2 + x^3 + x^4 + x^5 + \cdots$ involving infinitely many powers of x.

(e) determine the amount of money that needs to be deposited in a bank account in order to finance an annual payment of €1 in perpetuity, assuming that the bank account will forever pay fixed interest of

$i = 25\%$ on deposits at the end of each year. (Let $x = 1/(1+i)$ and note that €x earns interest of €1 after 1 year, €x^2 earns interest of €1 after two years, and so forth.)

4. [Trigonometric functions]
A lighthouse is located on a small island 3 km away from the nearest point P on a straight shoreline and its light beam makes one revolution every four minutes. The light beam passes point P at time $t = 0$ seconds.

   (a) Give a formula, valid for $0 \leq t < 60$, for the distance D (in km) between the point P and the point where the light beam hits the shore at time t seconds.

   (b) Evaluate $D(t)$ for $t = 0, 15, 30, 45$ seconds.

   (c) Sketch the graph of $D(t)$ for the time interval $0 \leq t \leq 45$.

   (d) Determine the average speed of the light beam on the shoreline over the time interval $t = 0$ to $t = 15$ seconds.

   (e) Determine the average speed of the light beam on the shoreline over the time interval $t = 15$ to $t = 30$ seconds.

   (*cf.* [James], Sec. 3.5, Problem 38)

5. [Exponential function]
An Essay on the Principle of Population, written by the Rev Thomas Robert Malthus and published in 1798, is one of the earliest and most influential books on population. It reasons that the size $P(t)$ of a population at a given time t is modelled by the equation

$$P(t) = Ae^{kt}$$

   where A and k are constants that depend on the population being modelled.

   Let us suppose that t is measured in years and that $t = 0$ corresponds to 1950.

   (a) Use the fact that the world population was 2560 million in 1950 to determine the constant A for the world population.

   (b) Use the fact that the world population was 3040 million in 1960 to determine the constant k for the world population.

(c) Use this *Malthusian model* to estimate the population of the world in 1993.

(*cf.* [Stewart], Sec. 3.8, Example 1)

6. [Limits of rational functions]
Aristotle had taught that heavy objects fall faster than lighter ones, in direct proportion to weight. Story has it that Galileo Galilei (1564-1642) dropped balls of the same material, but different masses, from the Leaning Tower of Pisa to demonstrate that their time of descent was independent of their mass. *Galileo's law* states that, ignoring air resistance, the distance $s(t)$ in meters travelled by a falling object after t seconds is given by

$$s(t) = 4.9t^2 \ .$$

Suppose that a ball is dropped from the roof of the Eiffel Tower, 300m above the ground.

(a) How far does the ball fall in the first 5 seconds?

(b) How long does it take the ball to reach the ground?

(c) How far does the ball travel in the interval from $t = 4$ s to $t = 5$ s?

(d) What is the average speed of the ball between $t = 4$ s and $t = 5$ s?

(e) Evaluate $\lim_{h\to 5} \frac{s(5)-s(5+h)}{h}$.

(f) What is the speed of the ball at $t = 5$ s?

(*cf.* [Stewart], Sec. 2.1, Example 3)

7. [Limits of rational functions]
Evaluate the following limits:

(a) $\displaystyle\lim_{x\to -3} \frac{x^2+5x+6}{x^2-4x-21}$

(b) $\displaystyle\lim_{h\to 0} \frac{(3+h)^2-9}{h}$

(c) $\displaystyle\lim_{x\to 3} \frac{\sqrt{x+6}-3}{x-3}$

(d) $\displaystyle\lim_{x\to \infty} \frac{2x^3-7x+4}{10x^3+2x+7}$.

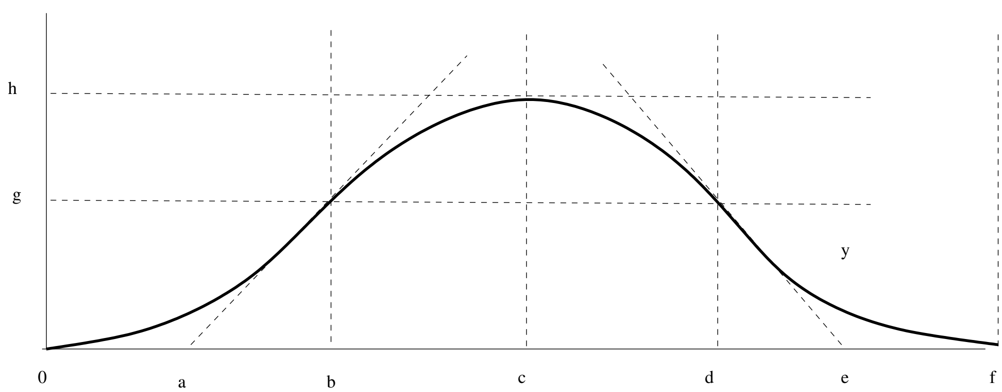
(*cf.* [Stewart], Sec. 2.3, 2.5, 2.6)

8. [Limits of trigonometric functions]
   Evaluate
   (a) $\lim_{\theta \to 0} \dfrac{\sin 6\theta}{\tan 7\theta}$
   (b) $\lim_{\theta \to 0} \dfrac{\cos\theta - 1}{\theta}$
   (c) $\lim_{x \to 0} \sin(2/x)$.

9. [Graphs]
   Sketch the graphs of the following functions.

$$f(x) = \frac{1}{x}, \quad g(x) = \frac{1}{x} - 2, \quad h(x) = \frac{1}{x-2}, \quad k(x) = \frac{1-x}{x-2}.$$

10. [Graphs] A particle travels along a straight line. Its distance from a fixed point on the line at time t is a continuous function $y(t)$ whose graph is illustrated (with horizontal t-axis). There are points of inflection at $(2, 2)$ and $(4, 2)$.



   (a) On which interval(s) is the particle accelerating (i.e. $y''(t) \geq 0$)?
   (b) On which interval(s) is the particle decelerating (i.e. $y''(t) \leq 0$)?
   (c) What is the maximum speed of the particle?
   (d) At what times(s) between $t = 1$ and $t = 5$ is the particle stationary?
   (e) How far has the particle travel between $t = 0$ and $t = 3$?

1.2.2 Differentiation

1. [Derivative of polynomial functions]
   Let $f(x) = x^4 - 6x^2 + 4$.

   (a) Find the derivative $f'(x)$.

   (b) Determine all values of x for which $f'(x) = 0$.

   (c) Find the points on the curve $y = x^4 - 6x^2 + 4$ where the tangent line is horizontal.

   (d) Find the equation of the tangent to the curve $y = x^4 - 6x^2 + 4$ at $x = 1$.

   (*cf.* [Stewart], Sec. 3.1, Example 6.)

2. [Derivative of trigonometric functions]
   Let $y = \cos t$.

   (a) Find the derivative dy/dx.

   (b) Find all values of t in the range $0 \leq t \leq 2\pi$ where the tangent to the curve $y = \cos t$ is horizontal.

   (c) Find the equation of the tangent to the curve $y = \cos t$ at $t = \pi/2$.

3. [Derivative of a sum]
   Let $f(x) = \sin x + 2\cos x$.

   (a) Find the derivative $f'(x)$.

   (b) What is the maximum slope of a tangent to the curve $y = \sin x + 2\cos x$?

   (c) What is the minimum slope of a tangent to the curve $y = \sin x + 2\cos x$?

4. [Derivative of a product]

   (a) Find the derivative of $f(x) = (1 + 2x)\sqrt{(x)}$.

   (b) Find a function $f(x)$ whose derivative is $f'(x) = \sin x \cos x$.

   (*cf.* [Stewart], Sec. 3.2, Example 2.)

5. [Derivative of a quotient]
   Find the derivative of the fuction $f(x) = \dfrac{x^2 + x - 2}{x^3 + 6}$.
   (*cf.* [Stewart], Sec. 3.2, Example 4.)

6. [Chain rule]
   Find the derivative of $y = (x^3 - 1)^{100}$.
   (*cf.* [Stewart], Sec. 3.4, Example 3.)

7. [Chain rule]
   Let $f(x) = \dfrac{1}{\sqrt[3]{x^2 + x + 1}}$. Finde $f'(x)$.
   (*cf.* [Stewart], Sec. 3.4, Example 4.)

8. [Chain rule]
   Find the derivative of $y = \cos(\sin(x))$.
   (*cf.* [Stewart], Sec. 3.4, Example 8.)

9. [Derivative of the exponential and logarithmic functions]
   (a) Find the derivative of $y = e^{\sin(x^2)}$.
   (b) Find the derivative of $f(x) = \ln(x^3 + 1)$.

   (*cf.* [Stewart], Sec. 3.4, Example 9 and Sec. 3.6, Example 1.)

10. [Implicit MA133-2/differentiation]
    Find y' if $y = x^3 + y^3 = 6xy$.
    (*cf.* Sec. 3.5, Example 2.)

1.2.3 Maxima, Minima & Related Rates

1. [maxima/minima]
   Let $f(x) = x^3 - 3x^2 + 1$.
   (a) Find the critical points of $f(x)$.
   (b) For each critical point decide whether it is a local maximum or a local minimum.
   (c) Find the minimum value of $f(x)$ on the interval $-1/2 \le x \le 4$.

(d) Find the maximum valule of $f(x)$ on the interval $-1/2 \leq x \leq 4$.

(*cf.* [Stewart], Sec. 4.1, Example 8)

2. [maxima/minima]
Find the absolute maximum and absolute minimum values of $f(x) = 3x^2 - 12x + 5$ on the interval $0 \leq x \leq 3$.

(*cf.* [Stewart], Sec. 4.1)

3. [maxima/minima]
A farmer wishes to fence off $900m^2$ of land adjacent to a road. It costs 40 Euro per metre to erect a fence adjacent to the road, but only 10 Euro per metre to erect a fence not adjacent to the road. Assuming the area to be fenced is rectangular, how long should the fence along the road be if the toal cost of all fencing is to be minimized?

(*cf.* [Stewart], Sec. 4.1)

4. [maxima/minima]
A box is to be made from a rectangular sheet of cardboard $70cm$ by $150cm$ by cutting equal squares out of the four corners and bending four flaps to make the sides of the box. (The box has no top.) What is the largest possible volume of the box?

(*cf.* [Stewart], Sec. 4.1)

5. [maxima/minima]
Find the maximum value of xy where x, y are real numbers satisfying

$$x^2 + \frac{y^2}{4} = 1.$$

(*cf.* [Stewart], Sec. 4.1)

6. [related rates]
A lighthouse is located on a small island 3 km away from the nearest point P on a straight shoreline and its light makes four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P ?

(*cf.* [Stewart], Sec. 3.9)

7. [related rates]
   An aircraft is flying horizontally at a speed of 600 km/h. How fast is the distance between the aircraft and a radio beacon increasing 1 minute after the aircraft passes 5km directly above the beacon?

   (*cf.* [Stewart], Sec. 3.9)

8. [related rates]
   At a certain instant the length of a rectangle is 16m and the width is 12m. The width is increasing at 3m/s. How fast is the length changing if the area of the recangle is not changing?

   (*cf.* [Stewart], Sec. 3.9)

9. [related rates]
   A rectangular water tank is being filled at a constant rate of 20 litres per second. The base of the tank is 1 metre wide and 2 metres long. How fast is the height of the water increasing?

   (*cf.* [Stewart], Sec. 3.9)

10. [related rates]
    At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 kmh and ship B is sailing north at 25 kmh. How fast is the distance between the ships changing at 4:00 PM?

    (*cf.* [Stewart], Sec. 3.9)

1.3 MA208 Quantitative techniques for business, Sem I

1. A university Mathematics Department runs a one-year MA degree in Mathematics and a one-year MSc degree in Mathematics. In both programmes students must study Algebra, Calculus and Geometry. The following table summarizes the number of staff hours required for each mathematics student per year, along with the total number of staff hours available per year.