MA208 Problem Sheet 5

1. Graph the following inequalities:

   (i) $6x - 3y > 18$

   (ii) $2x - 3y \leq 6$

2. A firm manufactures wood screws and metal screws. All the screws have to pass through a threading machine and a slotting machine. A box of wood screws requires 3 minutes on the slotting machine and 2 minutes on the threading machine. A box of metal screws requires 2 minutes on the slotting machine and 8 minutes on the threading machine. In a week. each machine is available for 60 hours.

   There is a profit of €10 per box on wood screws and €17 per box on metal screws.

   Formulate this problem as a linear programming problem given that the objective is to *maximise* profit.

   (Note: you are NOT asked to solve this problem.)

3. A manufacturing company makes two types of water skis, a trick ski and a slalom ski. The trick ski requires 6 labour-hours for fabricating and 1 labour-hour for finishing. The slalom ski requires 4 labour hours for fabricating and 1 labour-hour for finishing. The maximum labour-hours available per day for fabricating and finishing are 108 and 24, respectively.

   (i) If x is the number of trick skis and y is the number of slalom skis produced per day, write a system of linear inequalities that indicates appropriate constraints on x and y.

   (ii) Find the region of feasible solutions graphically for the number of each type of ski that can be produced.

   (iii) If the profit on a trick ski is €40 and the profit on a slalom ski is €30, how many of each type should be manufactured each day to realise a maximum profit? What is the maximum profit?

   (iv) Discuss the effect on the production schedule and the maximum profit if the profit on a slalom ski decreases to €25.

   (v) Discuss the effect on the production schedule and the maximum profit if the profit on a slalom ski increases to €45.

4. Ann and Margaret run a small business in which they work together making blouses and skirts. Each blouse takes 1 hour of Ann's time together with 1 hour of Margaret's time. Each skirt involves Ann for 1 hour and Margaret for half a hour. Ann has 7 hours available each day and Margaret has 5 hours each day. They could just make blouses or they could make some of each.

   Their first thought was to make the same number of each. But they get €8 profit on a blouse and only €6 on a skirt.

   (i) Formulate the problem as a linear programming problem.

   (ii) Solve the linear programming problem to calculate how many blouses and skirts they should make to maximise their profit. What is the profit?

MA208 Problem Sheet 4

1. 44000 people attended a match in Croke Park. The two ticket prices on the day were €30 and €20. The total receipts for the game came to €1.2 million. How many people paid the higher ticket price?

   Represent this problem by a system of linear equations and solve it.

2. An investor buys a total of 360 shares of two stocks. The price of one stock is €35 per share, while the price of the other stock is €45 per share. The investor spends a total of €15000. How many shares of each stock did the investor buy?

   Represent this problem by a system of linear equations and solve it.

3. €6000 is divided between two accounts, one paying 4% interest and the other paying 3% interest. At the end of one interest period, the interest earned by the 4% account exceeds the interest earned by the 3% account by €65. How much was invested in each account?

   Represent this problem by a system of linear equations and solve it.

4. Consider the matrices
$$A = \begin{pmatrix} 1 & 2 \\ 4 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -1 \\ -1 & 8 \end{pmatrix}.$$

   Calculate (i) $3A$, (ii) $3A - 2B$, (iii) $A^2 = A \cdot A$, (iv) $A \cdot B$ and (v) $B \cdot A$.

5. Find the inverse of the matrix $A = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$.

6. Show that $\begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$ is the inverse of $\begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix}$.

7. Use the Gauss-Jordan method to find the inverse of the matrix $A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 6 & -1 \\ 0 & 0 & 4 \end{pmatrix}$.

8. Consider the matrices
$$A = \begin{pmatrix} 3 & 2 & 6 \\ 4 & 3 & 4 \\ 4 & 2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & -6 & 10 \\ -4 & 15 & -12 \\ 4 & -2 & -1 \end{pmatrix}.$$

   Calculate the product $A \cdot B$.

9. A brewery produces Brown Ale, Dark Brown Ale and Porter. The following table summarizes the amount of malt, hops and yeast used to produce one keg of beer together with the total amount of these resources available per day.

Resource	Brown Ale	Dark Brown Ale	Porter	Daily available
Yeast	3 kg	2 kg	6 kg	250 kg
Hops	4 kg	3 kg	4 kg	220 kg
Malt	4 kg	2 kg	3 kg	170 kg

(a) Let x, y and z be the number of kegs of Brown Ale, Dark Brown Ale and Porter respectively produced daily. Write down a system of three linear equations which hold when all three resources are fully used.

(b) Find the values of x, y and z which ensure that the daily supply of hops, malt and yeast are fully used.

Hint: Use the result from Question 8.

10. A farmer has 200 acres of land suitable for cultivating crops A, B, and C. The cost per acre of cultivating crops A, B, and C is €40, €60, and €80, respectively. The farmer has €12,600 available for cultivation. Each acre of crop A requires 20 labour-hours, each acre of crop B requires 25 labour-hours, and each acre of crop C requires 40 labour-hours. The farmer has a maximum of 5950 labour-hours available. If she wishes to use all of her cultivatable land, the entire budget, and all the labour available, how many acres of each crop should she plant?

Note: Represent this problem by a system of linear equations, then use Gaussian Elimination or the inverse matrix method to solve this problem!

11. The annual returns on Sid Carrington's three investments amounted to €21,600: 6% on a savings account, 8% on mutual funds, and 12% on bonds. The amount of Sid's investment in bonds was twice the amount of his investment in the savings account, and the interest earned from his investment in bonds was equal to the dividends he received from his investment in mutual funds. Find how much money he placed in each type of investment.

Note: Represent this problem by a system of linear equations, then use Gaussian Elimination or the inverse matrix method to solve this problem!

MA208 Problem Sheet 3

1. Find the total amount due on a loan of €800 at 9% simple interest at the end of 4 months.

2. Suppose that you plan to buy a boat for €10,000 in three years. Given a simple interest rate of 4% per annum on a savings account, how much capital do you need to invest today in order to buy the boat in three years time?

3. Suppose a deposit is made into an account earning simple interest and after 2 years the balance is €27,600.

   (a) If the annual interest rate is 4%, how much was the original deposit?
   (b) If the original deposit was €25,000, what is the annual interest rate?

4. Aisha took a loan of €2400 with simple interest for as many years as the rate of interest. If she paid €864 as interest at the end of the loan period, what was the rate of interest?

5. Compare the interest earned after one year if €1000 is invested in an account at 8% compounded (a) daily and (b) continuously.

6. Mrs Smyth deposited €1,000 in a 2% account compounded continuously. In a second account, she deposited €500 in a 8% account compounded continuously.

   (a) When will the total amounts in both accounts be equal?
   (b) When will the total amount in the second accounts be 50% more than the total amount in the first account?

7. A person inherits the sum of €200,000. He wishes to invest some of this money so that in 15 years time his investment will have grown to €300,000. His bank has recommended an account which offers an interest rate of 12% per year compounded semi-annually. How much of his money should he invest?

8. A teenager plans to deposit €50 in a savings account at the end of each quarter for the next 6 years. Interest is earned at a rate of 8% per year compounded quarterly. What should his account balance be 6 years from now on? How much interest will he earn?

9. What is the value of an annuity at the end of 20 years if €2000 is deposited every year into an account earning 8.5% compounded annually? How much of this value is interest?

10. *Benefit Life* offers an ordinary annuity that earns 6.5% interest compounded annually. A person plans to make equal annual deposits into this account for 25 years and then make 20 equal annual withdrawals of €25,000, reducing the balance of the account to zero.

    (i) Compute the value of the fund based on the withdrawals required.
    (ii) Compute the amount of each deposit needed in order to maintain the fund.
    (iii) Compute the total interest earned over the entire 45 years.

11. **Probability Revision.** This question is from Exam 2015/16.

Assume that the oil depot near Dublin harbor imports oil from Russia and pays in rubles at the arrival of the tank ship. In order to hedge against the fluctuation of the ruble exchange rate, the oil depot buys stock options of Russian companies throughout the year, and sells them when the rubles for the incoming merchandise are needed. The shareholder value of the three companies selected by the oil depot, namely Gazprom, Norilsk Nickel and Aeroflot, also depends on the ruble exchange rate. Due to the different business fields the prices change in different ways summarized in the following table (in €per share).

Ruble exchange rate	Norilsk Nickel	Gazprom	Aeroflot
Sinking	€170	€−30	€10
Rising	€30	€160	€20
Stagnating	€−40	€−20	€90

Assume probabilities of 25% for a sinking, 25% for a rising and 50% for a stagnating ruble exchange rate.

(i) Which of the three stock options will give the maximum expected shareholder value? Explain your answer by calculating the expected shareholder value of all three stock options (in €per share).

(ii) What would be the value (in €per share) for the oil depot to know the development of the ruble exchange rate in advance ?

MA208 Problem Sheet 2

Q1. Your wardrobe consists of 5 shirts, 3 pairs of pants, and 17 bow ties. How many different outfits can you make?

Q2. A class consists of 6 girls and 7 boys. A team of four is to be formed. How many different teams can be formed if

   (i) There are no restrictions?

   (ii) There must be equal numbers of boys and girls?

   (iii) The team must be all female?

Q3. From a list of 15 preferred stocks recommended by your broker, you will select three to invest in. How many different ways can you select the three stocks from the 15 recommended stocks?

Q4. SMARTIE, a small software company would like to make 5 letter codes using all of the letters in the word SMARTIE. How many codes can be made from all the letters in this word?

Q5. Let $P(A)$ denote the probability of event A. Now let events A and B satisfy the following, $P(A) = P(B) = 0.7$ and $P(A \cap B) = 0.6$. Compute

- $P(A \cup B)$
- $P(\bar{A} \cap B)$

Q6. A bag contains 5 white marbles, 3 black marbles and 2 green marbles. In each draw, a marble is drawn from the bag and not replaced. In three draws, find the probability of obtaining white, black and green in that order.

Q7. Consider the probability distributions shown here.

x	-4	-3	-2	-1	0	1	2	3	4
$P(x)$	0.02	0.07	0.10	0.15	0.30	0.18	0.10	0.06	0.02

   (a) Calculate the mean μ, and the standard deviation σ.

   (b) Draw a graph of $P(x)$ in histogram form. Locate μ, $\mu - 2\sigma$ and $\mu + 2\sigma$ on the graph.

   (c) What is the probability that x is in the interval $(\mu - 2\sigma, \mu + 2\sigma)$?

Q8. In the old days, there was a probability of 0.8 of success in any attempt to make a telephone call. (This often depended on the importance of the person making the call, or the operator's curiosity!) Calculate the probability of having 7 successes in 10 attempts.

Q9. If electricity power failures occur according to a Poisson distribution with an average of 3 failures every twenty weeks, calculate the probability that there will not be more than one failure during a particular week.

Q10. In a card game with my friend, I pay a certain amount of money each time I lose. I win €4 if I draw a jack or a queen and I win €5 if I draw a king or ace from an ordinary pack of 52 playing cards. If I draw other cards, I lose. What should I pay so that we come out even?

Q11. (a) Paula tosses a coin four times and records the number X of heads. What is the probability that (i) $X = 0$, (ii) $X = 1$, (iii) $X = 2$, (iv) $X = 3$, (v) $X = 4$?

   (b) If X is odd then Paula pays Eoghan €$3X$. Otherwise Eoghan pays Paula €$3X$. What is the expected value of Paula's winnings on this game? Would you advise her to play this game?

MA208 Problem Sheet 1

Q1. The scores of 21 exam students are:

77, 88, 92, 57, 55, 73, 64, 89, 44, 76, 63, 75, 93, 56, 70, 79, 84, 51, 68, 66, 74

  (a) Present the data in a histogram using classes $0-9$, $10-19$, . . . , $90-99$.

  (b) Describe the shape of this histogram.

  (c) What is the median exam score?

Q2. Given the following data set:

$$
\begin{array}{cccccccccc}
55 & 49 & 37 & 57 & 46 & 40 & 64 & 35 & 73 & 62 \\
61 & 43 & 72 & 48 & 54 & 69 & 45 & 78 & 46 & 59 \\
40 & 58 & 56 & 52 & 49 & 42 & 62 & 53 & 46 & 81 \\
\end{array}
$$

  (a) Construct a stem and leaf plot for the above.

  (b) Compute the mean for the above.

  (c) Give a five point number summary for the above data.

  (d) Draw a boxplot for the above.

Q3. Consider the following data set:

$$
\begin{array}{cccccccccc}
55 & 49 & 37 & 57 & 46 & 40 & 64 & 35 & 73 & 62
\end{array}
$$

  (a) Compute the mean, harmonic mean and geometric mean for the above.

  (b) Compute the median.

  (c) Compute the range.

  (d) Find the standard deviation and coefficient of variation.

Q4. Define the terms *mean* and *median*. Give an example of a data set where the median is a better measure of the central tendency of the values - support your answer with reasons.

Q5. A shop selling soft drinks, records the number of sales during the morning period and during the afternoon period. The data recorded is given below,

$$\text{Morning: } \{70, 45, 80\}, \qquad \text{Afternoon: } \{60, 90, 30, 50, 35\}.$$

Compute the mean sales number for each period, and then use the weighted mean to compute the average number of sales overall.

Q6. Computers from a particular company are found to last on average for three years without any hardware malfunction, with a standard deviation of two months. At least what percent of the computers last between 31 months and 41 months?

Note: Use Chebyshev's inequality.