1. Sketch the points $(3, 0, 6)$, $(-2, -5, -8)$ and $(-1, 3, 0)$ on a single set of coordinate axes.

2. Sketch

   (a) $x + y = 2$ in $\mathbb{R}^2$

   (b) $x + y = 2$ in $\mathbb{R}^3$

3. Find an equation of the sphere that passes through the origin and whose center is $(1, 2, 3)$.

4. Show that the equation represents a sphere, and find its center and radius.

   (a) $x^2 + y^2 + z^2 + 8x - 6y + 2z + 17 = 0$

   (b) $2x^2 + 2y^2 + 2z^2 = 8x - 24z + 1$

5. Write inequalities to describe the region.

   (a) The region between the yz-plane and the vertical plane $x = 5$

   (b) The solid upper hemisphere of the sphere of radius 2 centered at the origin.

6. What is the relationship between the point $(4, 7)$ and the vector $\langle 4, 7 \rangle$? Illustrate with a sketch.

7. Let $A(4, 0, -2)$ and $B(4, 2, 1)$. Find a vector $\mathbf{a}$ with representation given by the directed line segment $\overrightarrow{AB}$. Draw $\overrightarrow{AB}$ and the equivalent representation starting at the origin.

8. Find the sum of the given vectors and illustrate geometrically.

   (a) $\langle -2, -1 \rangle$ and $\langle 5, 7 \rangle$

   (b) $\langle -1, 0, 2 \rangle$ and $\langle 0, 4, 0 \rangle$

9. If $\mathbf{a} = \langle 2, -4, 4 \rangle$ and $\mathbf{b} = \langle 0, 2, -1 \rangle$, find

   (a) $\mathbf{a} + \mathbf{b}$

   (b) $2\mathbf{a} + 3\mathbf{b}$

   (c) $|\mathbf{a}|$

   (d) $|\mathbf{a} - \mathbf{b}|$

10. Show that if $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are two- or three-dimensional vectors and c and d are scalars then

1. $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$
2. $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$
3. $\mathbf{a} + \mathbf{0} = \mathbf{a}$
4. $\mathbf{a} + (-\mathbf{a}) = \mathbf{0}$
5. $c(\mathbf{a} + \mathbf{b}) = c\mathbf{a} + c\mathbf{b}$
6. $(c + d)\mathbf{a} = c\mathbf{a} + d\mathbf{a}$
7. $(cd)\mathbf{a} = c(d\mathbf{a})$
8. $1\mathbf{a} = \mathbf{a}$

1. Show that $2i + 2j - k$ is perpendicular to $5i - 4j + 2k$

2. Find a vector perpendicular to the plane that passes through the points $p(1, 4, 6)$, $q(-2, 5, -1)$ and $r(1, -1, 1)$.

3. Use the scalar triple product to show that the vectors $A = < 1, 4, -7 >, B = < 2, -1, 4 >$ and $C = < 0, -9, 18 >$ are coplanar.

4. Find a unit vector that has the same direction as the given vector.

   (a) $\langle -4, 2, 4 \rangle$

   (b) $8\mathbf{i} - \mathbf{j} + 4\mathbf{k}$

5. Find a vector equation and parametric equations for the line.

   (a) The line through the point $(6, -5, 2)$ and parallel to the vector $\mathbf{i} + 3\mathbf{j} - \frac{2}{3}\mathbf{k}$

   (b) The line through the point $(0, 14, -10)$ and parallel to the line $x = -1 + 2t$, $y = 6 - 3t$, $z = 3 + 9t$.

6. Find parametric equations equations for the line.

   (a) The line through the origin and the point $(1, 2, 3)$

   (b) The line through the points $(6, 1, -3)$ and $(2, 4, 5)$

7. Find a vector equation and parametric equations for the line segment from $(10, 3, 1)$ to $(5, 6, -3)$.

8. Find the point at which the given lines intersect:

$$\mathbf{r} = \langle 1, 1, 0 \rangle + t \langle 1, -1, 2 \rangle$$

$$\mathbf{r} = \langle 2, 0, 2 \rangle + s \langle -1, 1, 0 \rangle$$

9. Find an equation of the plane.

   (a) The plane through the point $(-2, 8, 10)$ and perpendicular to the line $x = 1 + t, y = 2t, z = 4 - 3t$

   (b) The plane through the point $(4, -2, 3)$ and parallel to the plane $3x - 7z = 12$

10. Find the angle between the planes $x + y + z = 1$ and $x - 2y + 3z = 1$.

1. Find the limit.

(a) $\lim_{t\to 0}\left\langle \dfrac{e^t-1}{t}, \dfrac{\sqrt{1+t}-1}{t}, \dfrac{3}{1+t}\right\rangle$

(b) $\lim_{t\to 0}\left(e^{-3t}\mathbf{i}+\dfrac{t^2}{\sin^2 t}\mathbf{j}+\cos(2t)\mathbf{k}\right)$

2. Sketch the curve with the given vector equation. Indicate with an arrow the direction in which t increases.

(a) $\mathbf{r}(t)=\langle\cos(3t), t, \sin(3t)\rangle$

(b) $\mathbf{r}(t)=\langle\cos(3t), 3, \sin(3t)\rangle$

3. Find the derivative of the vector function.

(a) $\mathbf{r}(t)=\langle t\cos(3t), t^2, t\sin(3t)\rangle$

(b) $\mathbf{r}(t)=e^{-3t}\mathbf{i}+\mathbf{j}+e^{2t}\sin(4t)\mathbf{k}$

4. Find the unit tangent vector $\mathbf{T}(t)$ at the point with the given value of the parameter t.

(a) $\mathbf{r}(t)=\langle 4\sqrt{t}, t^2, t\rangle$, $\quad t=1$

(b) $\mathbf{r}(t)=\cos(t)\mathbf{i}+3t\mathbf{j}+2\sin(2t)\mathbf{k}$, $\quad t=0$

5. If $\mathbf{u}$ and $\mathbf{v}$ are differentiable vector functions, c is a scalar and f is a real-valued function show that

(a) $\dfrac{d}{dt}[\mathbf{u}(t)+\mathbf{v}(t)]=\mathbf{u}'(t)+\mathbf{v}'(t)$

(b) $\dfrac{d}{dt}[c\mathbf{u}(t)]=c\mathbf{u}'(t)$

(c) $\dfrac{d}{dt}[f(t)\mathbf{u}(t)]=f'(t)\mathbf{u}(t)+f(t)\mathbf{u}'(t)$

6. Evaluate the integral.

(a) $\displaystyle\int_0^1 \left(16t^3\mathbf{i}-9t^2\mathbf{j}+25t^4\mathbf{k}\right)\,dt$

(b) $\displaystyle\int_0^1 \left(\frac{4}{1+t^2}\mathbf{j}+\frac{2t}{1+t^2}\mathbf{k}\right)\,dt$

(c) $\displaystyle\int \left(e^t\mathbf{i}+2t\mathbf{j}+\ln t\mathbf{k}\right)\,dt$

(d) $\int (\cos(\pi t)\mathbf{i} + \sin(\pi t)\mathbf{j} + t\mathbf{k})\, dt$

7. Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = t\mathbf{i} + e^t\mathbf{j} + te^t\,\mathbf{k}$ and $\mathbf{r}(0) = \mathbf{i} + \mathbf{j} + \mathbf{k}$.

8. Find the length of the curve.

(a) $\mathbf{r}(t) = \langle 2\sin t, 5t, 2\cos t\rangle$, $\quad -10 \le t \le 10$

(b) $\mathbf{r}(t) = \langle 2t, t^2, \frac{1}{3}t^3\rangle$, $\quad 0 \le t \le 1$

(c) $\mathbf{r}(t) = \mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ $\quad 0 \le t \le 1$

(d) $\mathbf{r}(t) = 12t\mathbf{i} + 8t^{3/2}\mathbf{j} + 3t^2\mathbf{k}$ $\quad 0 \le t \le 1$

9. Reparametrize the curve $\mathbf{r}(t) = e^{2t}\cos(2t)\mathbf{i} + 2\mathbf{j} + e^{2t}\sin(2t)\mathbf{k}$ with respect to arc length measured from $(1, 2, 0)$ in the direction of increasing t.

10. Find the curvature of a circle of radius 3. What is the curvature of a circle of radius r?

11. Find the curvature of $\mathbf{r}(t) = \frac{1}{3}t^3\mathbf{i} + t^2\mathbf{j} + 2t\mathbf{k}$ (This question is computationally more challenging so do not worry if you cannot finish it up)

1. Find the domain and the range of the following functions and sketch the graphs.

(a) $f(x,y) = -4$

(b) $f(x,y) = y$

(c) $f(x,y) = 5x - 3y + 15$

(d) $f(x,y) = \sqrt{9 - x^2 - y^2}$

2. Decide if the following limits exist. Justify your answer.

(a) $\displaystyle \lim_{(x,y)\to(0.0)} \frac{xy^4}{x^2 + y^8}$

(b) $\displaystyle \lim_{(x,y)\to(0.0)} \frac{x^4 - y^4}{x^2 + y^2}$

(c) $\displaystyle \lim_{(x,y)\to(0.0)} \frac{xy}{x^2 + y^2}$

3. Determine the set of points at which the function is continuous.

(a) $f(x,y) = \dfrac{x - y}{1 + x^2 + y^2}$

(b) $f(x,y) = e^{x^2 y} + \sqrt{x + y^2}$

(c) $f(x,y) = \ln\left(x^2 + y^2 - 4\right)$

4. Find the partial derivatives of the function.

(a) $f(x,y) = 4y^4 - 5x^2y + 2$

(b) $f(x,y) = x^3 + \cos(xy)$

(c) $f(x,y) = x^y$

5. Use the chain rule to find the indicated partial derivatives.

(a) $z = x^2 + xy^3\ ,\ x = uv^2 + w^3\ ,\ y = u + ve^w\ ;\ \frac{\partial z}{\partial u} = ?\ ,\ \frac{\partial z}{\partial v} = ?\ ,\ \frac{\partial z}{\partial w} = ?$

(b) $R = ln(u^2 + v^2 + w^2)\ ,\ u = x + 2y\ ,\ v = 2x - y\ ,\ w = 2xy\ ;\ \frac{\partial R}{\partial x} = ?$

6. Find the equation of the tangent plane to the ellipsoid

$$\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$$

at the point $(-2, 1, -3)$.

7. Find the directional derivative of the function at the given point in the direction of the vector $\mathbf{v}$.

(a) $f(x,y) = x^2y^3 - 4y$ at point $(2,-1)$ in the direction of the vector $\mathbf{v} = 2\mathbf{i} + 5\mathbf{j}$

(b) $f(x,y) = 1 + 2x\sqrt{y}$ at point $(3,4)$ in the direction of the vector $\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}$

8. Find and classify the critical points of the function.

(a) $f(x,y) = 10x^2y - 5x^2 - 4y^2 - x^4 - 2y^4$

(b) $f(x,y) = x^4 + y^4 - 4xy + 1$